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TEM-POLARIZATION 2D-PHOTONIC CRYSTAL DESIGN USING FOURIER EXPANSION IN COMPLEX QUANTUM HILBERT SPACE

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ABSTRACT

Optical properties of photonic crystal (PC) structures have attracted much attention in the field of research area because of its ability to direct and control light propagation in compact device with feature sizes comparable to the wavelength of light. Therefore, from the early days of quantum mechanics, physicists have tried to understand the optical properties of photonic crystal in Hilbert space using Fourier expansion.

In this paper a theoretical study of two-dimensional (2D) planar optical photonic crystals is carried out; this optical photonic crystal consisting of a dielectric layer and dispersion less. This software was written in MATLAB to simulate and analysis the photonic bands for 2DPC of TEM polarizations. Fourier coefficients for the expansion of dielectric constant are calculated with refractive index $n_1 = 1$ and $n_2 = 3.45$.

KEYWORDS: 2D Photonic Crystal, Fourier expansion, Hilbert space, Photonic Band Gap

INTRODUCTION

The structures of photonic crystals (PCs) are constants with dielectric periodically modulated whose distribution follows a periodicity of the order of a fraction of the optical wavelength [1]. If electric permittivity depends on two spatial variables only, x and y, the PCto be two-dimensional (2D) [2,3]. PC is a material that is periodically modulated the index of refractive on a length scale comparable to the desired operation wavelength. It is named a crystal because the crystal is formed by a periodic arrangement of basic building blocks. In addition, the term photonic is added since photonic crystals are designed to affect the properties propagation of photons.

When wave propagation is enters into a material, it made most important affected,because some feature of interest of this wave is modulated. The coherent waves are scattering at the interfaces between different featured areas. In the case of photons this feature is index of refractive. Other systems propagating such as waves, sound or electrons within a semiconductor, these features are the Young's module and the electric potential respectively [4,5].

The optical photonic crystals have been the subject of intense investigation because of its ability to control the properties of photons [6,7]. Photonic crystals ability has led to mold the flow of light resulted in a variety wonderful fascinating optical phenomena, such as omnidirectional reflection, low loss bends, high-Q cavities, negative refraction, and the design of thermal emission [8].

Figure 1show the basic theoretical background of PC.Lord Rayleighwas first time studied the propagation of electromagnetic wave in periodic media in 1887. This is consistent with the one-dimensional (1D) PC, and he knew the fact that they have a narrow band gap prohibiting light propagation through the planes, whichdependent on the angle,

because of the different periodicities experienced by light propagating at cases of non-normal incidences, resulting a reflected color that differ sharply with the angle. Although, during the next century, multilayer films received more intensive study, in 1987, Yablonovitch and John joined the tools of classical electromagnetism and solid-state physics, that the concepts of omnidirectional photonic band gaps in 2D and 3D was introduced [9,10]. The generalization inspired the name Photonic Crystal was led to many developments in fabrication, theory, and application, from integrated optics to negative refraction to optical fibers that guide light in air [9,11].

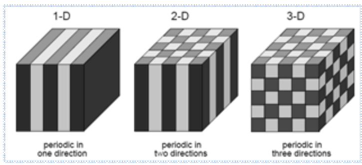


Figure1:One, two and three direction of photonic crystals periodic. The crystal material is periodicity structure [9,10].

2d Photonic Crystal Tem-Polarized Light

The electric field vector wave equation's for 2DPCsshown below [12]:

$$\begin{aligned}&\nabla\times\nabla\times\vec{E}(\vec{r})=k^2\varepsilon_r(\vec{r})\vec{E}(\vec{r})\\&\varepsilon(\vec{r})=\varepsilon(x,y)=\varepsilon_0\varepsilon_r(x,y)\\&\vec{E}(\vec{r})=\vec{E}(x,y,z)\end{aligned}\quad\ldots(1)$$

where $\varepsilon_r(x,y)$ isPC relative permittivity function's. The vector of the electric field is a function of three spatial coordinates system. For a TM-polarized field i.e.: $\vec{E}(\vec{r})=E_z(\vec{r})\hat{z}$, the divergence of the field is always zero. In the normal propagation case, i.e. (angle of 90^0 with z axis), after that, the electric field, $\vec{E}_z(\vec{r})$, is shown as a function of x and y, i.e. [12]:

$$\begin{cases}\nabla^2E_z(x,y)+k_0^2\varepsilon_r(x,y)E_z(x,y)=0\\ \nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\end{cases}\ldots(2)$$

Using variablesseparation method, electric field can be written as below [6]:

$$\begin{aligned}&\frac{\partial^2}{\partial x^2}\chi(x)+\frac{\omega^2}{c^2}\left[X(x)+\beta\right]\chi(x)=0\\&\frac{\partial^2}{\partial y^2}\psi(y)+\frac{\omega^2}{c^2}\left[Y(y)-\beta\right]\psi(y)=0\end{aligned}\quad\ldots(3)$$

Equation (2) reduces to two separate equations.One 2D structure has been divided into two separate 1D structures, one in the vertical and the other in the horizontal direction.

Such as above procedure to reduce the wave equation into two separate differential equations on the lengthof each coordinate, and we are trying to do this for magnetic TE-polarized field. Wave equation for two dimensions TE-polarization can be considered below [6]:

$$\begin{aligned}&\nabla\times\left[\frac{1}{\varepsilon_r(\vec{r})}\nabla\times\vec{H}(\vec{r})\right]=k_0^2\vec{H}(\vec{r})\ldots(4)\\&\varepsilon(\vec{r})=\varepsilon(x,y)=\varepsilon_0\varepsilon_r(x,y)\end{aligned}$$

$\varepsilon_r(x,y)$ isPC relative permittivity function's. Vector of the magnetic field is a function of the three spatial coordinates system. For TE polarized light only the normal component of the field would be nonzero. Below, the derive ofmagnetic field component equation[6]:

$$\begin{aligned}&\nabla\eta\times(\nabla\times\vec{H})-\eta\nabla^2\vec{H}=k_0^2\vec{H}(\vec{r})\\&\eta=\frac{1}{\varepsilon_r(\vec{r})}\end{aligned}\quad\ldots(5)$$

$$\frac{\partial}{\partial x}\eta\frac{\partial}{\partial x}H_z+\frac{\partial}{\partial y}\eta\frac{\partial}{\partial y}H_z+\eta\frac{\partial^2}{\partial x^2}H_z+\eta\frac{\partial^2}{\partial y^2}H_z+\eta\frac{\partial^2}{\partial z^2}H_z=-k_0^2H_z\cdots(6)$$

Using the Fourier transformation for solution, so, the solution write as, $H_z(x,y,z)=A(x,y)e^{-i\gamma z}$, where γ coefficient corresponds to the wave vector angle's with respect to *z* axis [6]:

$$\frac{\partial}{\partial x}\eta\frac{\partial}{\partial x}A+\frac{\partial}{\partial y}\eta\frac{\partial}{\partial y}A+\eta\frac{\partial^2}{\partial x^2}A+\eta\frac{\partial^2}{\partial y^2}A=A(\gamma^2\eta-k_0^2)\cdots(7)$$

Assuming*x* and *y* are field components, the product of two distinct functions of *x* and *y*, become, $A(x,y)=\chi(x)\psi(y)$ and [6]:

$$\frac{1}{\chi(x)}\frac{\partial^2}{\partial x^2}\chi(x)+\frac{1}{\psi(y)}\frac{\partial^2}{\partial y^2}\psi(y)+\left[\frac{1}{\eta}\frac{\partial}{\partial x}\eta\right][\frac{1}{\chi(x)}\frac{\partial}{\partial x}\chi(x)]+\left[\frac{1}{\eta}\frac{\partial}{\partial y}\eta\right][\frac{1}{\psi(y)}\frac{\partial}{\partial y}\psi(y)]=(\gamma^2-k_0^2/\eta)\ldots(8)$$

If the wave of incident propagates (angle= 90^0*z* axis ($\gamma=0$)) in the plane of PC, and the permittivity to be a sum of two terms, each being single functions of *x*and *y*, the magnetic field will take the form in equation (9). Thus,the set of TE-polarization wave equation in a two dimension photonic crystal for in-plane propagation found as [6]:

$$\Rightarrow\begin{cases}\frac{\partial^2}{\partial x^2}\chi(x)-\frac{1}{X(x)+Y(y)}\frac{\partial}{\partial x}X\left[\frac{\partial}{\partial x}\chi(x)\right]+[k_0^2X(x)-\beta_2]\chi(x)=0\ldots(9)\\\frac{\partial^2}{\partial y^2}\psi(y)-\frac{1}{X(x)+Y(y)}\frac{\partial}{\partial y}Y\left[\frac{\partial}{\partial y}\psi(y)\right]+[k_0^2Y(y)+\beta_2]\psi(y)=0\end{cases}$$

HILBERT SPACE FOURIER EXPANSION METHOD

Now, in the case of TM modes in 2D PC, the study has the two dimensionsHelmholtz equation [2]:

$$-\left(\frac{\partial^2\psi}{\partial^2x}+\frac{\partial^2\psi}{\partial^2y}\right)=\eta n^2(x,y)\psi\cdots(10)$$

Wherespectral parameter η and index of refractive $n(x)$ is a continuous positive function [2]:

$$n(x+m_1a_1+m_2a_2)=n(x),\quad x\in R^2,\quad m_1,m_2\in Z\ldots(11)$$

Seek ψ as a solutions bounded nontrivial of the Helmholtz (equation (10)) which is the Laplacian distribution $\nabla^2\psi$ is also borderedin R^2. Therefore, the solutions take Bloch representation [2]:

$$\psi(x)=e^{ik.x}\phi(x)\ldots(12)$$

where, ϕ satisfies the periodicity condition:

$\phi(x+m_1a_1+m_2a_2)=\phi(x), \quad x\in R^2, \quad m_1,m_2\in Z$, and the vector $k=t_1b_1+t_2b_2$ for certain integers t_1,t_2,b_1 and b_2 being the reciprocal vectors basis. Substituting (12) into (10) we get: $-\nabla^2\phi(x)-2ik.\nabla\phi(x)+\|k\|^2\phi(x)=\eta n(x)^2\phi(x)$.

After construct an orthonormal basis of the complex Hilbert space consisting of those functions in $L^2(A)$ that satisfy the above condition.Writing: $B=\{t_1b_1+t_1b_1:(t_1,t_2)\in Z^2\}$, the orthonormal basis is $\{m(A)^{-1/2}\varphi_t : t\in Z^2\}$, where: $\varphi_t(x)=e^{i(t_1b_1+t_2b_2).x}, t=(t_1,t_2)\in Z^2, \quad m(A)=\det A$.

Each basis function satisfies the periodicity condition: $\varphi_t(x+m_1a_1+m_2a_2)=\varphi_t(x), \quad (m_1,m_2)\in Z^2, t\in Z^2$, as well as the conjugation symmetry: $\overline{\varphi_t(x)}=\varphi_{-t}(x), \quad t\in Z^2$.

The computed of the expansion coefficients ϕ_t in $\phi(x)=\sum_{t\in Z^2}\phi_t\varphi_t(x)$ shown as follows:

$$\phi_t=m(A)^{-1}\int_A dx\phi(x)\overline{\varphi_t(x)}=m(A)^{-1}\int_A dx\phi(x)\varphi_{-t}(x)$$

Observe that: $-\nabla^2\varphi_t(x)=\|t_1b_1+t_2b_2\|^2\varphi_t(x)=4\pi^2t^T(A^TA)^{-1}t\varphi_t(x)$, where t is the column vector with integer elements t_1,t_2.

Expanding the squared refractive index $n(x)$ and an arbitrary ϕ in Hilbert space: $n(x)^2=\sum_{t\in Z^2}n_t\varphi_t(x), \quad \phi(x)=\sum_{t\in Z^2}\phi_t\varphi_t(x)$, where: $\int_A dxn(x)^4=m(A)\sum_{t\in Z^2}|n_t|^2$, and using $\varphi_t(x)\varphi_s(x)=\varphi_{t+s}(x)$ for $t,s\in Z^n$ whereas, Z^2 to be an additive group,

$$n(x)^2\phi(x)=\sum_{t\in Z^2}\left(\sum_{s\in Z^2}n_{t-s}\phi_s\right)\varphi_t(x)$$

As a result, the Helmholtz (equation (1)) with $\psi(x)=e^{ikx}\phi(x)$ for some given $k=\tau_1b_1+\tau_2b_2\in B$ (with $\tau=(\tau_1,\tau_2)\in Z^2$) written as [2]:

$$4\pi^2t^T(A^TA)^{-1}t\phi_{t-\tau}=\|t_1b_1+t_2b_2\|^2\phi_{t-\tau}=\lambda\sum_{s\in R^2|}n_{t|-s}\phi_s \quad \ldots(13)$$

Where: $t\in Z^2$. Equation (13) solved for $\{\phi_t\}_{t\in Z^2}$ in complex Hilbert space $\ell^2(Z^2)$.

This system written as below [2]:

$$4\pi^2(t+\tau)^T(A^TA)^{-1}(t+\tau)\phi_t=\|(t_1+\tau_1)b_1+(t_2+\tau_2)b_n\|^2\phi_{t-\tau}=\lambda\sum_{s\in R^2}n_{t+\tau-s}\phi_s \quad \ldots(14)$$

Where: $M=4\pi^2 diag([t+\tau]^T(A^TA)^{-1}[t+\tau])_{t\in Z^2}$ is an unbounded diagonal matrix with nonnegative entries.

T, a multi-index Toeplitz matrix, $\{n(x)^2 : x\in A\}$ is the multi-index spectrum.The compact subset of $(0,+\infty)$, show the condition $n(x)$ as a continuous function of $x\in A$.

Simulation Result and Discussion

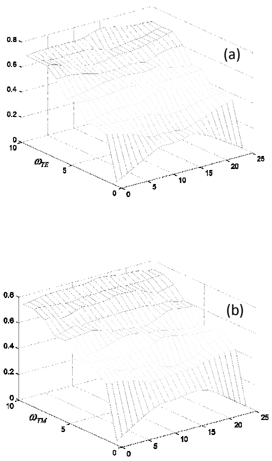
This simulation results design 1D photonic crystal and calculate the statistical analysis measurements with plots the photonic bands for 2DPC. Proceed with fields for TE and TM modes. The results below are got after following these steps:

Found the total number of plane waves used in Fourier expansions.

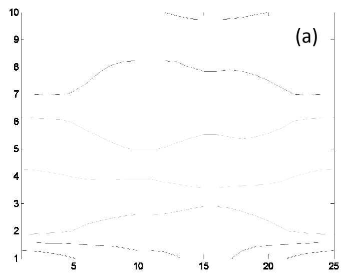
- Calculate the matrix of Fourier coefficients of dielectric function.
- Calculate the points along the perimeter of irreducible Brillouin zone.
- Calculate the diagonal matrices.
- Solve the eigenvalue problems for TEM polarization.

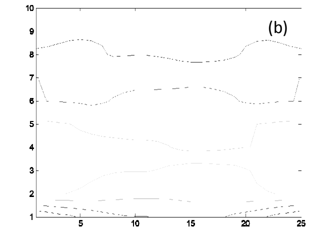
Figures 2 and Figures 3 represent the digital matrix as a wireframe surface for TE and TM polarization photonic crystal respectively. The TE mode statistical analysis: mean=0.4684, median=0.4833, mode=4.346E-005 and the standard deviation (STD)=0.2112. Whereas, mean=0.5218, median=0.5425, mode=4.953E-005 and STD=0.2156 for TM polarization mode. However we can see the more importanceof polarization effects in two dimensions systems after compere above statistical.

So that, in order to understand how to build the structure of the optical photonic band;we studied a 2D system. Although the sample presented in this paper is 2D photonic crystal is easier to explain. Calculations were limited to the case of Fourier expansion in Hilbert space for TE and TM modes. The relationship between the frequency and the wave vector (dispersion relation) in vacuum for free photons is: $w=ck$, where c is the light speed in vacuum.



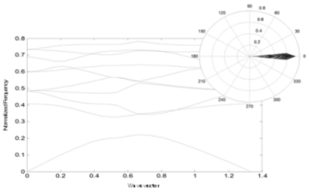
Figures 2:(a) Display a photonic crystal TE polarization digital matrix as a wireframe surface for three variables matrices defining values of surface crystal components. (b) Display a photonic crystal TM polarization digital matrix as a wireframe surface for three variables matrices defining values of surface crystal components.



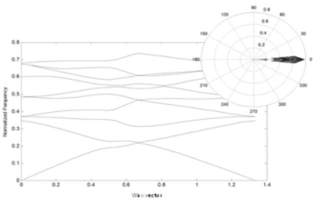


Figures 3:(a) Display isolines of a photonic crystal TE polarization. A surface represented by a digital matrixplotted variables. One variable (ω_{TE})-plot contour lines of the elements of a matrix, interpreting values as heights with respect to the x-y plane. Matrix must be at least 2-by-2 with three variables. (b) Display isolines of a photonic crystal TM polarization. A surface represented by a digital matrixplotted variables. One variable (ω_{TM})-plot contour lines of the elements of a matrix, interpreting values as heights with respect to the x-y plane. Matrix must be at least 2-by-2 with three variables.

The modeled structure behind Figures 4 and Figures 5 are a triangular lattice where the radius is r = 0.35. This structure presents the TE and TM polarization photonic bands for 2D PC consisting of cylinders and circular cross-section and infinite height arranged in a triangular lattice. The corner plots represent the vectors as arrows emanating from the origin on circular grid inside the photonic crystal. Two variables compass graph having m arrows, where m is the number of the two elements variables. The tip of each arrow is at a point relative to the base. This syntax is equivalent to compass real and imaginary elements.



Figures 4: The TE modes photonic crystal band structure. The right corner plots represent the vectors as arrows emanating from the origin on circular grid inside the TE photonic crystal.



Figures 5: The TM modes photonic crystal band structure. The right corner plots represent the vectors as arrows emanating from the origin on circular grid inside the TM photonic crystal.

CONCLUSIONS

The theoretical modeling and simulations are presented. At the beginning the background theory, which is necessary to understand photonic crystals presented. Two dimensional photonic crystals are investigated by using both Fourier expansion and Hilbert space method. The dispersion relation of the PCs is obtained by using MATLAB.

In this paper, finding a satisfactory analyticalmethod for the spectrum bandwith the mathematical properties of two dimensions optical photonic crystals were studied.The structure of the optical band gives us information about the electromagnetic radiation propagationproperties inside the optical photonic crystal and when the plotof energy states as a function of propagation direction is available, the structure is representation.

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